МИНИСТЕРСТВО ОБРАЗОВАНИЯ РЕСПУБЛИКИ БЕЛАРУСЬ

БЕЛОРУССКИЙ ГОСУДАРСТВЕННЫЙ УНИВЕРСИТЕТ

ФАКУЛЬТЕТ ПРИКЛАДНОЙ МАТЕМАТИКИ И ИНФОРМАТИКИ

ФУНКЦИОНАЛЬНЫЙ АНАЛИЗ

Домашняя работа №3
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1.

$$x_0 = (4, -1, 2, 0, \dots); L = \{x | \sum_{k=1}^{\infty} \frac{x_k}{2^k} = x_1 - 2x_2 = 0\}$$

Решение

$$\begin{cases} x_1 \frac{1}{2} + x_2 \frac{1}{4} + \dots + x_k \frac{x_k}{2^k} + \dots = 0 \\ x_1 - 2x_2 = 0 \end{cases} \quad (1)$$

$a = \left(\frac{1}{2}, \frac{1}{4}, \dots, \frac{1}{2^k}, \dots\right)$
$b = (1, -2, 0, \dots, 0, \dots)$
$(x_0, a) = \alpha(a, a) + \beta(a, b) = 2$
$(x_0, b) = \alpha(a, b) + \beta(b, b) = 6$
$(a, a) = \sum_{k=1}^{\infty} \frac{x_k}{2^{2k}} = \frac{1}{3}$
$(b, b) = 1 + 4 = 5$
$(a, b) = \frac{1}{2} - 2 * \frac{1}{4} = 0$
$\alpha = 6, \quad \beta = \frac{6}{5}$
$x = x_0 - (\alpha a + \beta b)$
Ответ
$x = \left(-\frac{1}{5}; -\frac{1}{10}; \frac{5}{4}; \dots; -\frac{3}{2^k}; \dots\right)$

2.

$$x_0 = (-2, 1, 0, 2, 0, -18, 0, \dots); L = \{x | \sum_{k=1}^{\infty} \frac{x_k}{3^k} = x_1 - x_4 + 9x_6 = 0\}$$

Решение
$a = \left(\frac{1}{3}, \frac{1}{9}, \dots, \frac{1}{3^k}, \dots\right)$
$b = (1, 0, 0, -1, 0, 9, 0 \dots, 0, \dots)$
$(x_0, a) = \alpha(a, a) + \beta(a, b) = -\frac{5}{9}$
$(x_0, b) = \alpha(a, b) + \beta(b, b) = -166$
$(a, a) = \sum_{k=1}^{\infty} \frac{x_k}{3^{2k}} = \frac{1}{8}$
$(a, b) = \frac{1}{3}$ $(b, b) = 1 + 1 + 81 = 83$
$\alpha = 0, \quad \beta = -2$
$x = x_0 - (\alpha a + \beta b)$
Ответ
$x = (0, -1, 0, \dots, 0, \dots)$

3.

$$x_0 = (1, 1, 0, 0, \dots); L = \{x | x_1 + x_3 = x_2 - x_4 = 0\}; \ P_k = \frac{1}{2^k}$$

Решение
$a = (2, 0, 8, 0, \dots)$
$b = (0, 4, 0, -16, 0, \dots)$
$(x_0, a) = \alpha(a, a) + \beta(a, b) = 1$
$(x_0, b) = \alpha(a, b) + \beta(b, b) = 1$
$(a, a) = 4 * \frac{1}{2} + 64 * \frac{1}{8} = 10$ $(b, b) = 16 * \frac{1}{4} + 256 * \frac{1}{16} = 20$
$(a, b) = 0$
$\alpha = 1/20, \quad \beta = 1/20$
$x = x_0 - (\alpha a + \beta b)$
Ответ
$x = (4/5, 4/5, -4/5, 4/5, 0, \dots, 0, \dots)$

4.

$$x_0 = (1, 1, 1, 0, \dots); L = \{x | \sum_{k=1}^{\infty} \frac{x_k}{3^k} = x_2 - 3x_3 = 0\}$$

Решение
$a = (1/3, 1/9, 1/27, 0, \dots)$
$b = (0, 1, -3, 0 \dots)$
$(x_0, a) = \alpha(a, a) + \beta(a, b) = 13/27$
$(x_0, b) = \alpha(a, b) + \beta(b, b) = -2$
$(a, a) = 1/8$ $(b, b) = 1 + 9 = 10$
$(a, b) = \frac{1}{9} - 3 * \frac{1}{27} = 0$
$\alpha = 104/27, \quad \beta = -1/5$
$x = x_0 - (\alpha a + \beta b)$
Ответ
$x = (1 - \frac{104}{3^4}, \frac{6}{5} - \frac{104}{3^5}, \frac{2}{5} - \frac{104}{3^6}, -\frac{104}{3^7}, \dots)$

5.

$$x_0 = (1, 1/2, 1/4, \dots, 1/2^k, \dots); L = \{x | x_1 - x_2 = x_3 + 4x_5 = 0\}$$

Решение
$a = (1, -1, 0, \dots)$
$b = (0, 0, 1, 0, 4, 0 \dots)$
$(x_0, a) = \alpha(a, a) + \beta(a, b) = 1/2$
$(x_0, b) = \alpha(a, b) + \beta(b, b) = 1/2$
$(a, a) = 1 + 1 = 2$ $(b, b) = 1 + 16 = 17$
$(a, b) = 0$
$\alpha = 1/2, \quad \beta = 1/68$
$x = x_0 - (\alpha a + \beta b)$
Ответ
$x = (1/2, 1, 1/4, 7/68, 1/16, -47/64, 1/2^6 \dots)$

6.

$$x_0 = (1, 1, 1, 0, \dots); L = \{x | x_1 - x_3 = x_2 + x_5 = 0\}; \;\; P_k = 2^k;$$

Решение
$a = (1/2, 0, -1/8, \dots)$
$b = (0, 1/4, 0, 0, 1/32, 0 \dots)$
$(x_0, a) = \alpha(a, a) + \beta(a, b) = 0$
$(x_0, b) = \alpha(a, b) + \beta(b, b) = 1$
$(a, a) = \frac{1}{2} * \frac{1}{4} + \frac{1}{8} * \frac{1}{64} = \frac{5}{8}$ $(b, b) = \frac{1}{4} * \frac{1}{16} + \frac{1}{32} * \frac{1}{1024} = \frac{9}{32}$
$(a, b) = 0$
$\alpha = 0, \quad \beta = 32/9$
$x = x_0 - (\alpha a + \beta b)$
Ответ
$x = (1, 1/9, 1, 0, -1/9, 0, \dots)$

7.

$$x_0 = (1, 1, 1, 0, \dots); L = \{x | \sum_{k=3}^{\infty} \frac{x_k}{5^k} = x_1 - 3x_2 = 0\};$$

Решение
$a = (0, 0, 1/5^3, \dots)$
$b = (1, -3, 0 \dots)$
$(x_0, a) = \alpha(a, a) + \beta(a, b) = 1/5^3$

$(x_0, b) = \alpha(a, b) + \beta(b, b) = -2$
$(a, a) = 1/5^4 24$ $(b, b) = 10$
$(a, b) = 0$
$\alpha = 120, \quad \beta = -1/5$
$x = x_0 - (\alpha a + \beta b)$
Ответ
$x = (6/5, 2/5, 1/25, -24/5^3, -24/5^4, \dots)$

8.

$$x_0 = (1, 1, 0, 1, 0, \dots); L = \{x | \sum_{k=3}^{\infty} \frac{x_k}{2^k} = x_1 + x_2 = 0\};$$

Решение
$a = (0, 0, 1/2^3, 1/2^4 \dots)$
$b = (1, 1, 0 \dots)$
$(x_0, a) = \alpha(a, a) + \beta(a, b) = 1/2^4$
$(x_0, b) = \alpha(a, b) + \beta(b, b) = 2$
$(a, a) = 1/48$ $(b, b) = 1 + 1 = 2$
$(a, b) = 0$
$\alpha = 3, \quad \beta = 1$
$x = x_0 - (\alpha a + \beta b)$
Ответ
$x = (0, 0, -3/8, 13/16, -3/25, \dots)$

9.

$$x_0 = (1/2, 1/4, 1/8, \dots); L = \{x | \sum_{k=1}^{\infty} \frac{x_k}{3^k} = x_1 - 9x_5 = 0\};$$

Решение
$a = (1/3, 1/9, 1/3^3, \dots)$
$b = (1, 0, 9, 0 \dots)$
$(x_0, a) = \alpha(a, a) + \beta(a, b) = 1/5$
$(x_0, b) = \alpha(a, b) + \beta(b, b) = -5/8$
$(a, a) = 1/8$ $(b, b) = 1 + 81 = 82$
$(a, b) = 0$
$\alpha = 8/5, \quad \beta = -5/656$
$x = x_0 - (\alpha a + \beta b)$
Ответ
$x = (-506/19680, 13/180, 27/20, \dots)$

10.

$$x_0 = (-3, 7, 0, 1, 0, \dots); L = \{x | \sum_{k=1}^{\infty} \frac{x_k}{2^k} = x_1 + x_2 = 0\};$$

Решение
$a = (1/2, 1/4, 1/8, \dots)$
$b = (1, 1, 0 \dots)$
$(x_0, a) = \alpha(a, a) + \beta(a, b) = 5/16$
$(x_0, b) = \alpha(a, b) + \beta(b, b) = 4$
$(a, a) = 1/3$ $(b, b) = 1 + 1 = 2$ $(a, b) = 3/4$
$\alpha = -114/5, \quad \beta = 211/20$
$x = x_0 - (\alpha a + \beta b)$
Ответ
$x = (-43/20, 43/20, -114/40, 194/80, \dots)$